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POSTAL BOOK PACKAGE 2025

ELECTRICAL ENGINEERING

CONVENTIONAL Practice Sets

CONTENTS

ELECTROMAGNETIC THEORY

1 CHAPTER Vector Analysis

Q.1 Given point, $P(9, -12, 15)$ is in Cartesian system. Express P in cylindrical and spherical systems.

Solution

$$\rho = \sqrt{x^2 + y^2} = \sqrt{(9)^2 + (-12)^2} = 15$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\left[\frac{(-12)}{9}\right] = -53.13^\circ$$

P is cylindrical format will be as, $P = (15, -53.13^\circ, 15)$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{(9)^2 + (-12)^2 + (15)^2} = 21.213$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1}\left[\frac{15}{15}\right] = 45^\circ$$

P in spherical format will be as, $P = (21.213, 45^\circ, -53.13^\circ)$

Q.2 Let $\vec{H} = 5\rho\sin\phi\,\hat{a}_\rho - \rho z\cos\phi\,\hat{a}_\phi + 2\rho\hat{a}_z$ A/m. At point $P(2, 30^\circ, -1)$ find:

(a) a unit vector along $\vec{H}$.

(b) the component of $\vec{H}$ parallel $\hat{a}_x$.

(c) the component of $\vec{H}$ normal to $\rho = 2$.

(d) the component of $\vec{H}$ tangential to $\phi = 30^\circ$.

Solution:

At P, $\rho = 2, \phi = 30^\circ, z = -1$

$$\vec{H} = 10\sin 30^\circ\,\hat{a}_\rho + 2\cos 30^\circ\hat{a}_\phi - 4\hat{a}_z = 5\hat{a}_\rho + 1.732\,\hat{a}_\phi - 4\hat{a}_z \text{ A/m}$$

(a) Unit vector along $\vec{H}$,

$$\hat{a}_H = \frac{5\hat{a}_\rho + 1.732\,\hat{a}_\phi - 4\hat{a}_z}{\sqrt{5^2 + 1.732^2 + 4^2}} = 0.7538\,\hat{a}_\rho + 0.2611\hat{a}_\phi - 0.603\hat{a}_z$$

(b) $H_x = H_\rho\cos\phi - H_\phi\sin\phi = 5\rho\sin\phi\cos\phi - \rho z\cos\phi\sin\phi$

or, P at $\rho = 2,\quad \phi = 30^\circ, z = -1$

$$H_x = H_x\,\hat{a}_x = (10\sin 30^\circ\cos 30^\circ + 2\sin 30^\circ\cos 30^\circ)\,\hat{a}_x = 5.196\,\hat{a}_x \text{ A/m}$$

(c) Normal to $P = 2$ is $\vec{H}_\rho = \vec{H}_\rho\,\hat{a}_\rho$

i.e. $\vec{H}_n = 0.7538\,\hat{a}_\rho$ A/m

(d) Tangential to $\phi = 30^\circ$

$$H_t = H_\rho\,\hat{a}_\rho + H_z\,\hat{a}_z = 0.7538\,\hat{a}_\rho - 0.603\,\hat{a}_z \text{ A/m}$$

Q.3 E and F are vector fields given by $\vec{E} = 2x\,\hat{a}_x + \hat{a}_y + yz\hat{a}_z$ and $\vec{F} = xy\,\hat{a}_x - y^2\hat{a}_y + xyz\hat{a}_z$. Determine:

(a) $|E|$ at (1, 2, 3)

(b) The component of $\vec{E}$ along $\vec{F}$ at (1, 2, 3)

(c) A vector perpendicular to both $\vec{E}$ and $\vec{F}$ at (0, 1, –3) whose magnitude is unity.

Solution:

(a)
$$\vec{E} = 2x\,\hat{a}_x + \hat{a}_y + yz\,\hat{a}_z$$

At point (1, 2, 3)
$$\Rightarrow \quad \vec{E} = 2\hat{a}_x + \hat{a}_y + 6\hat{a}_z$$
$$|\vec{E}| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41} = 6.403$$

(b)
$$\vec{F} = xy\,\hat{a}_x - y^2\,\hat{a}_x + xyz\,\hat{a}_z$$

At (1, 2, 3),
$$\vec{F} = 2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z$$

$\therefore$ The component of $\vec{E}$ along $\vec{F}$

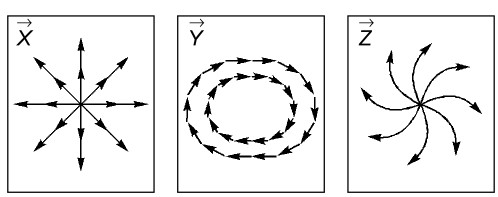
$$\vec{E}_F = (\vec{E}\cdot a_F)\hat{a}_F = \frac{(\vec{E}\cdot\vec{F})}{|\vec{F}|}\bar{a}_F = \frac{36}{56}(2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) = 1.286\hat{a}_x - 2.571\hat{a}_y + 3.857\hat{a}_z$$

(c) At (0, 1, –3)
$$\vec{E} = 0\hat{a}_x + \hat{a}_y - 3\hat{a}_z$$
$$\vec{F} = 0\hat{a}_x - \hat{a}_y + 0\hat{a}_z$$

$$E\times\vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = -3\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$$

$$a_{E\times F} = \pm\frac{\vec{E}\times\vec{F}}{|\vec{E}\times\vec{F}|} = \pm\,\hat{a}_x$$

Q.4 The figures show diagrammatic representations of vector fields $\vec{X}, \vec{Y}$ and $\vec{Z}$, respectively. What can you comment about these diagrams?



Solution:

$\vec{X}$ is going away so $\vec{\nabla}\cdot\vec{X} \neq 0$.

$\vec{Y}$ is moving circulator direction so $\vec{\nabla}\times\vec{Y} \neq 0$.

$\vec{Z}$ has circular rotation so $\vec{\nabla}\times\vec{Z} \neq 0$.

Q.5 Find the divergence of vector field, $\vec{V}(x,y,z) = -(x\cos xy + x)\,\hat{i} + (y\cos xy)\,\hat{j} + (\sin z^2 + x^2 + y^2)\,\hat{k}$.

Solution:

$$\vec{V}(x, y, z) = -(x\cos xy + y)\hat{i} + (y\cos xy)\hat{j} + [\sin(z)^2 + (x^2) + (y^2)]\hat{k}$$
$$\text{Divergence} = \nabla\cdot V$$
$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$
$$= -\cos xy + x(\sin xy)y + \cos xy - y\sin(xy)x + 2z\cos z^2 = 2z\cos z^2$$